

Decision 2 Linear Programming Answers

5(a)	Introducing slack variables	M1																														
	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left;"><i>P</i></th> <th style="text-align: left;"><i>x</i></th> <th style="text-align: left;"><i>y</i></th> <th style="text-align: left;"><i>z</i></th> <th style="text-align: left;"><i>r</i></th> <th style="text-align: left;"><i>s</i></th> <th style="text-align: left;">value</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-3</td> <td>-2</td> <td>-4</td> <td>0</td> <td>0</td> <td>0</td> </tr> <tr> <td>0</td> <td>1</td> <td>4</td> <td style="border: 1px solid black; border-radius: 50%; padding: 2px;">2</td> <td>1</td> <td>0</td> <td>8</td> </tr> <tr> <td>0</td> <td>2</td> <td>7</td> <td>3</td> <td>0</td> <td>1</td> <td>21</td> </tr> </tbody> </table>	<i>P</i>	<i>x</i>	<i>y</i>	<i>z</i>	<i>r</i>	<i>s</i>	value	1	-3	-2	-4	0	0	0	0	1	4	2	1	0	8	0	2	7	3	0	1	21	A2	3	-1 EE
<i>P</i>	<i>x</i>	<i>y</i>	<i>z</i>	<i>r</i>	<i>s</i>	value																										
1	-3	-2	-4	0	0	0																										
0	1	4	2	1	0	8																										
0	2	7	3	0	1	21																										
(b)	Choosing correct pivot in <i>z</i> -column	M1		and perhaps dividing by 2																												
	<table style="width: 100%; border-collapse: collapse;"> <tbody> <tr> <td>1</td> <td>-1</td> <td>6</td> <td>0</td> <td>2</td> <td>0</td> <td>16</td> </tr> <tr> <td>0</td> <td style="border: 1px solid black; border-radius: 50%; padding: 2px;">$\frac{1}{2}$</td> <td>2</td> <td>1</td> <td>$\frac{1}{2}$</td> <td>0</td> <td>4</td> </tr> <tr> <td>0</td> <td>$\frac{1}{2}$</td> <td>1</td> <td>0</td> <td>$-\frac{3}{2}$</td> <td>1</td> <td>9</td> </tr> </tbody> </table>	1	-1	6	0	2	0	16	0	$\frac{1}{2}$	2	1	$\frac{1}{2}$	0	4	0	$\frac{1}{2}$	1	0	$-\frac{3}{2}$	1	9	M1		row operations							
1	-1	6	0	2	0	16																										
0	$\frac{1}{2}$	2	1	$\frac{1}{2}$	0	4																										
0	$\frac{1}{2}$	1	0	$-\frac{3}{2}$	1	9																										
		A1	3	correct																												
(c)(i)	Need to use <i>x</i> – column for pivot Choosing correct pivot	M1 A1																														
	<table style="width: 100%; border-collapse: collapse;"> <tbody> <tr> <td>1</td> <td>0</td> <td>10</td> <td>2</td> <td>3</td> <td>0</td> <td>24</td> </tr> <tr> <td>0</td> <td>1</td> <td>4</td> <td>2</td> <td>1</td> <td>0</td> <td>8</td> </tr> <tr> <td>0</td> <td>0</td> <td>-1</td> <td>-1</td> <td>-2</td> <td>1</td> <td>5</td> </tr> </tbody> </table>	1	0	10	2	3	0	24	0	1	4	2	1	0	8	0	0	-1	-1	-2	1	5	M1 A1 A1	5	row operations top row third row							
1	0	10	2	3	0	24																										
0	1	4	2	1	0	8																										
0	0	-1	-1	-2	1	5																										
(ii)	Yes optimal No negative values in top row	B1✓ E1	2																													
Total			13																													

5(a)	$3x+7y \leq 33$ $x+2y \leq 10$ $2x+7y \leq 26$	M1		One correct inequality, or all using <																																																															
		A1	2	All correct																																																															
(b)(i)	Compare $\frac{33}{3}, \frac{10}{1}, \frac{26}{2}$ Choose smallest positive value \Rightarrow pivot = 1	E1																																																																	
		E1	2																																																																
(ii)	<table style="border-collapse: collapse; margin-bottom: 10px;"> <thead> <tr> <th style="padding: 2px 5px;"><i>P</i></th> <th style="padding: 2px 5px;"><i>x</i></th> <th style="padding: 2px 5px;"><i>y</i></th> <th style="padding: 2px 5px;"><i>r</i></th> <th style="padding: 2px 5px;"><i>s</i></th> <th style="padding: 2px 5px;"><i>t</i></th> <th style="padding: 2px 5px;">Value</th> </tr> </thead> <tbody> <tr> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">-1</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">40</td> </tr> <tr> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">-3</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">3</td> </tr> <tr> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">10</td> </tr> <tr> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">$\frac{2}{3}$</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">-2</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">6</td> </tr> </tbody> </table> <p style="margin-left: 20px;">next <i>y</i> pivot on $\frac{2}{3}$</p> <table style="border-collapse: collapse; margin-bottom: 10px;"> <tbody> <tr> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">$3\frac{1}{3}$</td> <td style="padding: 2px 5px;">$\frac{1}{3}$</td> <td style="padding: 2px 5px;">42</td> </tr> <tr> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">$-2\frac{1}{3}$</td> <td style="padding: 2px 5px;">$-\frac{1}{3}$</td> <td style="padding: 2px 5px;">1</td> </tr> <tr> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">$2\frac{1}{3}$</td> <td style="padding: 2px 5px;">$-\frac{2}{3}$</td> <td style="padding: 2px 5px;">6</td> </tr> <tr> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">$-\frac{2}{3}$</td> <td style="padding: 2px 5px;">$\frac{1}{3}$</td> <td style="padding: 2px 5px;">2</td> </tr> </tbody> </table>	<i>P</i>	<i>x</i>	<i>y</i>	<i>r</i>	<i>s</i>	<i>t</i>	Value	1	0	-1	0	4	0	40	0	0	1	1	-3	0	3	0	1	2	0	1	0	10	0	0	$\frac{2}{3}$	0	-2	1	6	1	0	0	0	$3\frac{1}{3}$	$\frac{1}{3}$	42	0	0	0	1	$-2\frac{1}{3}$	$-\frac{1}{3}$	1	0	1	0	0	$2\frac{1}{3}$	$-\frac{2}{3}$	6	0	0	1	0	$-\frac{2}{3}$	$\frac{1}{3}$	2	M1 A1 A1		Row operation Correct one row (<i>other than pivot row</i>) All correct
<i>P</i>	<i>x</i>	<i>y</i>	<i>r</i>	<i>s</i>	<i>t</i>	Value																																																													
1	0	-1	0	4	0	40																																																													
0	0	1	1	-3	0	3																																																													
0	1	2	0	1	0	10																																																													
0	0	$\frac{2}{3}$	0	-2	1	6																																																													
1	0	0	0	$3\frac{1}{3}$	$\frac{1}{3}$	42																																																													
0	0	0	1	$-2\frac{1}{3}$	$-\frac{1}{3}$	1																																																													
0	1	0	0	$2\frac{1}{3}$	$-\frac{2}{3}$	6																																																													
0	0	1	0	$-\frac{2}{3}$	$\frac{1}{3}$	2																																																													
		M1																																																																	
		m1 A1 A1	7	Row operation Correct one row (<i>other than pivot row</i>) All correct (condone multiples of given rows) (maximum 6 if <i>y</i> -pivot used first)																																																															
(iii)	No negative number in top row $P_{\max} = 42$ $x = 6$ $y = 2$	E1 B1✓ B1✓		ft if M3 scored and optimum reached																																																															
			3																																																																
Total			14																																																																

3(a)	P	x	y	z	s	t	Value					
	1	-5	-8	-7	0	0	0	M1		SCA		
	0	3	2	1	1	0	12	A2	3	-1 EE		
	0	2	4	5	0	1	16					
b(i)	$\frac{12}{2} = 6; \frac{16}{4} = 4$ and $4 < 6$							E1	1			
	(ii)	1	-1	0	3	0	2	32	M1		using 4 as pivot and possibly dividing third row by 4	
0		2	0	$-1\frac{1}{2}$	1	$-\frac{1}{2}$	4	A1		top row correct		
0		$\frac{1}{2}$	1	$1\frac{1}{4}$	0	$\frac{1}{4}$	4	A1		second row correct; may have		
	choice of pivot from x-column							M1			pivot = 2 identified and used	
	1	0	0	$2\frac{1}{4}$	$\frac{1}{2}$	$1\frac{3}{4}$	34					
	0	1	0	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	2	m1		row operations		
	0	0	1	$1\frac{5}{8}$	$-\frac{1}{4}$	$\frac{3}{8}$	3	A1	6	correct or scaled up		
(iii)	Max $P = 34$							B1✓				
	$x = 2, y = 3, z = 0$							B1	2			all correct
(iv)	Yes - no negative values in first row							E1✓	1			no - if negatives in top row
Total									13			

4(a)	$x + 2y \leq 36$ $x + y \leq 20$ $4x + y \leq 39$	M1		One correct, or all inequalities with <																																			
		A1	2	All correct																																			
(b)(i)	Choosing 2 as pivot	M1		And perhaps dividing second row by 2																																			
	<table border="1"> <thead> <tr> <th><i>P</i></th> <th><i>x</i></th> <th><i>y</i></th> <th><i>s</i></th> <th><i>t</i></th> <th><i>u</i></th> <th>value</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>$-\frac{1}{2}$</td> <td>0</td> <td>$2\frac{1}{2}$</td> <td>0</td> <td>0</td> <td>90</td> </tr> <tr> <td>0</td> <td>$\frac{1}{2}$</td> <td>1</td> <td>$\frac{1}{2}$</td> <td>0</td> <td>0</td> <td>18</td> </tr> <tr> <td>0</td> <td>$\frac{1}{2}$</td> <td>0</td> <td>$-\frac{1}{2}$</td> <td>1</td> <td>0</td> <td>2</td> </tr> <tr> <td>0</td> <td>$3\frac{1}{2}$</td> <td>0</td> <td>$-\frac{1}{2}$</td> <td>0</td> <td>1</td> <td>21</td> </tr> </tbody> </table>	<i>P</i>	<i>x</i>	<i>y</i>	<i>s</i>	<i>t</i>	<i>u</i>	value	1	$-\frac{1}{2}$	0	$2\frac{1}{2}$	0	0	90	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	18	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	1	0	2	0	$3\frac{1}{2}$	0	$-\frac{1}{2}$	0	1	21	m1		Row operations
<i>P</i>	<i>x</i>	<i>y</i>	<i>s</i>	<i>t</i>	<i>u</i>	value																																	
1	$-\frac{1}{2}$	0	$2\frac{1}{2}$	0	0	90																																	
0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	18																																	
0	$\frac{1}{2}$	0	$-\frac{1}{2}$	1	0	2																																	
0	$3\frac{1}{2}$	0	$-\frac{1}{2}$	0	1	21																																	
		A1		One row correct																																			
		A1	4	All rows correct (condone multiples of rows)																																			
(ii)	Negative value in top row \Rightarrow optimum not yet reached	E1	1																																				
(c)(i)	New pivot (<i>x</i> -column, 3rd row)	M1		And perhaps multiplying by 2																																			
	<table border="1"> <thead> <tr> <th><i>P</i></th> <th><i>x</i></th> <th><i>y</i></th> <th><i>s</i></th> <th><i>t</i></th> <th><i>u</i></th> <th>value</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>0</td> <td>2</td> <td>1</td> <td>0</td> <td>92</td> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> <td>1</td> <td>-1</td> <td>0</td> <td>16</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> <td>-1</td> <td>2</td> <td>0</td> <td>4</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> <td>3</td> <td>-7</td> <td>1</td> <td>7</td> </tr> </tbody> </table>	<i>P</i>	<i>x</i>	<i>y</i>	<i>s</i>	<i>t</i>	<i>u</i>	value	1	0	0	2	1	0	92	0	0	1	1	-1	0	16	0	1	0	-1	2	0	4	0	0	0	3	-7	1	7	m1		Row operations
<i>P</i>	<i>x</i>	<i>y</i>	<i>s</i>	<i>t</i>	<i>u</i>	value																																	
1	0	0	2	1	0	92																																	
0	0	1	1	-1	0	16																																	
0	1	0	-1	2	0	4																																	
0	0	0	3	-7	1	7																																	
		A1		One row correct																																			
		A1	4	All rows correct																																			
(ii)	Optimum value reached $\left. \begin{array}{l} P = 92, x = 4, y = 16 \\ s = 0, t = 0, u = 7 \end{array} \right\}$	E1		(Or not? – if their tableau wrong)																																			
		B1✓		FT 3 values																																			
		B1	3	CSO (final tableau must be correct)																																			
	Total		14																																				